

Box 2

WHAT DO OPTION RISK-NEUTRAL DENSITY ESTIMATES TELL US ABOUT THE EURO/DOLLAR EXCHANGE RATE?

Risk-neutral densities (RNDs) provide an estimate of the probability that market participants attach to future price developments. They are derived from the option prices of a given asset, and under certain assumptions can provide a distribution of outcomes on the basis of which a quantitative risk assessment can be made. In the case of exchange rates, they are an important tool in assessing the likelihood of sharp movements – a key risk for financial stability – and the evolution of this likelihood over time. This box introduces RND estimates for the euro/dollar exchange rate, briefly describes how they are constructed, explains how they can be interpreted and discusses the information they provide at the current juncture.

RNDs can be used to build probability distributions on the basis of two assumptions. The first is that investors are risk-neutral. The second is that options are available for all strikes (that is, the values relevant for exercising the options). With these assumptions, the ratio between the difference in two option prices relative to the difference in option strikes provides the information needed for calculating the relative probabilities of the exchange rate reaching these two levels.

For example, the discrepancy between the prices of two sell options with very close strike prices (e.g. one at USD/EUR 1.2000 and another at USD/EUR 1.2001) would be the probability-weighted difference in option pay-offs. Having a continuum of option prices, one could calculate the whole probability distribution.¹ However, in practice, option prices are available only for a limited number of strikes. Therefore, one needs to estimate – rather than calculate – the overall distribution implied by these prices.

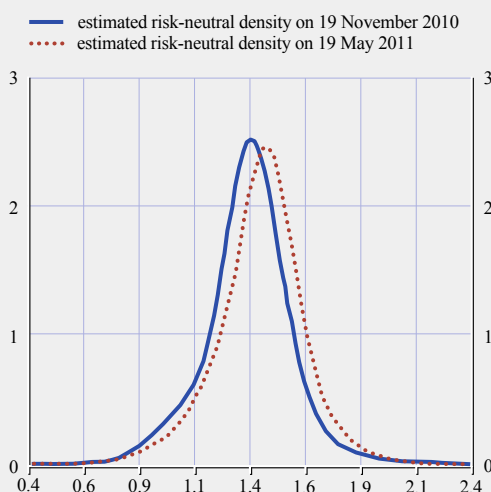
RNDs are not reliable tools for prediction purposes and they are not used for forecasting exchange rates. Rather, their value lies in the information they convey about sentiment on the foreign exchange market: estimated RNDs can be used, for example, as a tool to track changes in market sentiment since the peak of the sovereign debt crisis. By comparing RND estimates based on data up to two different dates, one can assess how foreign exchange market expectations have changed, both in terms of the central estimate and in terms of variance, skewness (larger likelihood of appreciation or of depreciation) and kurtosis (i.e. the probability of large appreciations/deprecations) of the distribution (see the chart).

The chart displays the 12-month-horizon RND for the USD/EUR exchange rate, estimated with data up to 19 November 2010 (the cut-off date of the December 2010 FSR) and up to 19 May 2011. The distribution was skewed to the right on both dates, which means that a euro appreciation was viewed as more probable than a depreciation. On 19 May 2011 the distribution was centred on a higher value of the euro. This is not surprising since the spot exchange rate appreciated between the two dates. To evaluate the likelihood attached by the market to the tail risk of an extreme euro appreciation, we look at the estimated probability of the USD/EUR exchange rate appreciating by more than 26.5%, which is the largest year-on-year appreciation recorded so far for this currency pair. This decreased from 3.1% to 2.2% at a 12-month horizon between 19 November 2010 and 19 May 2011 (see table below). Furthermore, looking at the moments of the 12-month-ahead distribution, the variance, skewness and kurtosis all decreased slightly over the period, indicating that the market attached a lower probability to euro appreciation than before, while the tails of the distribution were thinner. The estimated probability that the USD/EUR exchange rate would appreciate by more than 26.5% is shown in the table for various horizons, from one month to one year. This probability was virtually zero on both dates at the shorter horizons and it decreased at the longer horizons of 6 and 12 months, from 0.5% and 3.1% to 0.3% and 2.2% respectively.

¹ This has been shown by S. Ross, "Options and efficiency", *Quarterly Journal of Economics*, 90, 1976, pp. 75-89, and subsequently developed by D. Breeden and R. Litzenberger, "Prices of State Contingent Claims Implicit in Option Prices", *Journal of Business*, 51, 1978, pp. 621-652. See also ECB, "The information content of option prices during the financial crisis", *Monthly Bulletin*, February 2011.

Estimated USD/EUR risk-neutral density

(based on 12-month-horizon option prices; USD/EUR exchange rate on the x-axis)



Sources: Bloomberg and ECB calculations.

Overall, the shape of the estimated RND functions indicates that markets still attach a higher probability to an appreciation of the euro against the US dollar than a depreciation, as shown by the thicker right tail of the distribution. However, the probability of an extreme appreciation has decreased. While exchange rate movements have been affected by many factors since November 2010, in particular by improving macroeconomic developments on average in the euro area, the change in the position and shape of the estimated USD/EUR risk-neutral density also suggests that the policy actions taken at the height of the crisis contributed to stabilising expectations in the exchange rate market. This, in turn, indicates a decreased likelihood of disruption to the balance sheets of those agents that are exposed to currency risk.

Probability of the USD/EUR exchange rate appreciating by 26.5% in May 2011 and in November 2010

(based on one, three, six and twelve-month-horizon option prices; percentages)

Horizon	19 November 2010	19 May 2011
1 month	0.00	0.00
3 months	0.00	0.00
6 months	0.50	0.30
12 months	3.10	2.20
<i>Memo item: spot rate</i>	<i>1.37</i>	<i>1.43</i>

Sources: Bloomberg and ECB calculations.